## Maths Written Calculation Policy

This document maps Fawbert and Barnard Infant School's and Reedings Junior School's pathway to the required written formal calculation methods as outlined in the National Curriculum (2013) Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division.

It links the key concrete experiences with pictorial and abstract representations and focusses on the vocabulary used at each stage. This supports pupils to move with confidence and deep conceptual understanding through each strand of calculation.

While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies and known facts that form the basis of all calculations. A range of mental strategies are developed throughout ESSENTIALmaths: the program both schools use to teach maths.

Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved.

## Concrete, Pictorial and Abstract

## Concrete manipulatives

Concrete manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways.

## Pictorial (including jottings)

The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper unders tanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives.

## Abstract

The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

## Vocabulary

Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.


## Vocabulary

Addition, add, more, and, make, sum, total, equal, altogether, double, near double, half, halve, one more/two more/ten more/one hundred more, how many more to make ...? how many more is ... than ...? how much more is ...?


Addition, add, more, and, make, sum, total, equal, altogether, double, near double, half, halve, one more/two more/ten more/one hundred more, how many more to make ...? how many more is ... than ...? how much more is ...?

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## Year 3 NC Statement: Add numbers with up to three digits, using formal written methods of columnar addition with regrouping of ones.

## Concrete

e.g. $247+135=$


12 ones is regrouped into 1 ten and 2 ones


Other pictorial models, such as bar models, can support understanding of the concept.

382
247

Abstract
e.g. $247+135=$


$$
247+135=382
$$

NB: children may cross out the regrouped numbers underneath once they have been added.

## Vocabulary

Addition, add, more, and, make, sum, total, equal, altogether, double, near double, half, halve, one more/two more/ten more/one hundred more, how many more to make ...? how many more is ... than ...? how much more is ...?

The sum of ... ones and ... ones is ... ones.
The sum of ... tens and ... tens is ...tens.
The sum of ... hundreds and ... hundreds is ... hundreds.
So, ... $+\ldots$ is equal to ... hundreds, ... tens and ... ones, which is ... .

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## Vocabulary

Addition, add, more, and, make, sum, total, equal, altogether, double, near double, half, halve, one more/two more/ten more/one hundred more, how many more to make ...? how many more is ... than ...? how much more is ...?

The sum of ... ones and ... ones is ... ones. This is regrouped into ... tens and ... ones.
The sum of ... tens and ... tens is ...tens. This is regrouped into ... hundreds and ... tens.
The sum of ... hundreds and ... hundreds is ... hundreds.
So, ... $+\ldots$ is equal to ... hundreds, ... tens and ... ones, which is ...


$$
276+56=332
$$

NB: children may cross out the regrouped numbers once they have been added.


Once pupils have fully understood and rehearsed regrouping within formal column addition of 3-digit numbers, this learning continues to be rehearsed and applied throughout Years 4, 5 and 6, including to 4-digit numbers, larger numbers, decimal numbers, money and measures and problem solving with missing numbers.


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| Year 2 | NC Statement: Subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two, two-digit numbers. With regrouping. |  |  |
| :---: | :---: | :---: | :---: |
| Concrete | Concrete | Pictorial <br> "I can see that there aren't enough ones for me to take away 6 ones without regrouping. <br> Regroup one ten into ten ones. There are now 6 tens and 13 ones." | Abstract |
| Vocabulary <br> How many are left/left over? How many have gone? One less/two less/ten less/one hundred less, how many fewer is ... than ...? How much less is ...? Difference between, equals, is the same as, number bonds/pairs/facts, tens boundary. |  |  |  |


| Year 3 | NC Statement: Subtract numbers with up to three digits, using formal written methods of columnar subtraction. <br> No regrouping. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concrete $\text { e.g. } 345-124=$ |  |  | Pictorial$\text { e.g. } 345-124=$Hundreds Tens Ones <br> $\square$ \|ld $\cdot$ •000 <br> $\square$   <br> Other pictorial models, such as bar and cherry models, can support understanding of the concept. <br> $345-124=$ |  |  | Abstract $\text { e.g. } 345-124=$ |  |
| Hundreds | Tens | Ones |  |  |  | 345 |  |
|  | $\\|_{\text {\# }}^{\text {- }}$ | - XXX |  |  |  | $-124$ | $\cdots$ |
|  |  |  |  |  |  | $345-124=221$ | - |
| Vocabulary <br> How many are left/left over? How many have gone? One less/two less/ten less/one hundred less, how many fewer is ... than ...? How much less is ...? Difference between, equals, is the same as, number bonds/pairs/facts, tens boundary. <br> ... ones take away ... ones leaves ... ones. <br> ... tens take away ... tens leaves ... tens. <br> ... hundreds take away ... hundreds leaves ... hundreds. <br> So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is ... |  |  |  |  |  |  |  |


\section*{| Year 3 | NC Statement: Subtract numbers with up to three digits, using formal written methods of |
| :--- | :--- | columnar subtraction with regrouping tens into ones.}



"I can see that there aren't enough ones for me to take away 4 ones without regrouping. Regroup one ten into ten ones. There are now ten ones and zero ones. 10 ones take away 4 ones leaves six ones. 7 tens take away 2 tens leaves 5 tens. So, $80-24$ is equal to 5 tens and 6 ones, which is 56 ."

Abstract
e.g. $345-124=$


- 24

56
$80-24=56$

## Vocabulary

How many are left/left over? How many have gone? One less/two less/ten less/one hundred less, how many fewer is ... than ...? How much less is ...? Difference between, equals, is the same as, number bonds/pairs/facts, tens boundary.

I can see that there aren't enough ones for me to take away ... ones without regrouping.
Regroup one ten into ten ones.
There are now ... tens and ... ones.
... ones take away ... ones leaves ... ones.
... tens take away ... tens leaves ... tens.
So, ... $-\ldots$ is equal to... tens and ... ones, which is ... .



| Year 3 | $\begin{array}{l}\text { NC Statement: Subtract numbers with up to three digits, using formal written methods of } \\ \text { columnar subtraction with regrouping hundreds into tens }\end{array}$ |
| :--- | :--- | columnar subtraction with regrouping hundreds into tens.



Abstract
e.g. 323-141 =

## ${ }^{2} z^{1} 23$

- 141

182

$$
323-141=182
$$

## Vocabulary

How many are left/left over? How many have gone? One less/two less/ten less/one hundred less, how many fewer is ... than ...? How much less is ...? Difference between, equals, is the same as, number bonds/pairs/facts, tens boundary.
... ones take away ... ones leaves ... ones.
I can see that there aren't enough tens for me to take away ... tens without regrouping.
Regroup one hundred into ten hundreds.
There are now ... hundreds and ... tens.
... tens take away ... tens leaves ... tens.
... hundreds take away ... hundreds leaves ... hundreds
So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is ....

| Year 3 | $\begin{array}{l}\text { NC Statement: Subtract numbers with up to three digits, using formal written methods of } \\ \text { columnar subtraction with regrouping hundreds and tens. }\end{array}$ |
| :--- | :--- |



Abstract
e.g. $404-226=$


$$
404-226=178
$$

Children are encouraged to keep workings clear to ensure accurate answers.

## Vocabulary

How many are left/left over? How many have gone? One less/two less/ten less/one hundred less, how many fewer is ... than ...? How much less is ...? Difference between, equals, is the same as, number bonds/pairs/facts, tens boundary.

I will need to regroup...

- one hundred into ten tens. I now have ... hundreds and ... tens.
- one ten into ten ones. I now have ... tens and ... ones.

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Once pupils have fully understood and rehearsed regrouping within formal subtraction, this learning continues to be rehearsed and applied throughout Years 4,5 and 6 , including to multi-digit, decimal numbers, money and measures.


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| Year 3 | NC Statement: Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. <br> Short multiplication - no regrouping. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Concrete$\text { e.g. } 12 \times 3$ |  | Pictorial <br> e.g. $12 \times 3$ <br> Pictorial methods, such as this informal cherry model, allow children to break the strategy into smaller, more manageable parts. | Abstract $\text { e.g. } 12 \times 3$ <br> The children |  |
| Tens | Ones |  | 12 multiplying the |  |
| ппппп | $\square \square$ |  | 36 | ㄷ |
|  |  |  | $12 \times 3=36$ | $\bigcirc$ |
| This would also be discussed as repeat addition of $12(12+12+12)$ or 12 three times. |  |  | " 3 groups of 2 ones is 6 ones. 3 groups of 1 ten is 3 tens. 3 tens added 6 ones is 36 . The product of 12 and 3 is 36 ." | $\stackrel{0}{+}$ |
| Vocabulary multiply/times \# by \#, \# lots of \# equals..., \# groups of \# equals..., repeated addition, product, multiplied by, array ... groups of ... ones is ... ones. <br> ... groups of ... tens is ... tens. <br> ... tens added to ... ones is ... . <br> The product of ... and ... is ... . |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## Vocabulary

NC Statement: Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Short multiplication - regrouping ones into tens.



$$
24 \times 3=72
$$

" 3 groups of 4 ones is 12 ones. I can regroup the 12 ones into 1 ten and 2 ones. 3 groups of 2 tens is 6 tens. 1 ten added to 6 tens is 7 tens. The product of $24 \times 3$ is 72 ."
multiply/times \# by \#, \# lots of \# equals..., \# groups of \# equals..., repeated addition, product, multiplied by, array, regrouping
... groups of ... ones is ... ones.
I can regroup the ... ones into ... ten(s) and ... one(s).
... groups of ... tens is ... tens.
... ten(s) added to ... is ...
The product of ... and ... is ... .


## Vocabulary

multiply/times \# by \#, \# lots of \# equals..., \# groups of \# equals..., repeated addition, product, multiplied by, array, regrouping
... groups of ... ones is ... ones. (Do I need to regroup?)
... groups of ... tens is ... tens. (Do I need to regroup?)
... groups of ... hundreds is ... hundreds. (Do I need to regroup?)
(... hundreds can be regrouped to ... thousands and ... hundreds)

The product of ... and ... is ... .



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## Vocabulary

divide, share, group, divided by, equal sharing, divisible by, remainder

I am sharing ... ones into ... equal groups.
There are ... ones in each group.
I have ... one(s) remaining.
The quotient is ... with ... remainders.

## Vocabulary

divide, share, group, divided by, equal sharing, equal groups, divisible by, remainder, quotient

First, I am sharing ... tens into ... equal groups.
There are ... tens in each group.
I have ... ten(s) remaining.
Then, I am sharing ... ones into ... equal groups.
There are ... ones in each group.
I have ... one(s) remaining.
The quotient is ... with ... remainders.
"First, I am sharing 8 tens into 2 equal groups. There are 4 tens in each group. I have zero tens remaining. Then, I am sharing 4 ones into 2 equal groups. There are 2 ones in each group. I have zero ones remaining. The quotient is 42 with no remainders."

Abstract
e.g. $84 \div 2$
\(\begin{array}{r}4 <br>

2\)| 4 | 2 |
| :---: | :---: |
|  | 8 |
|  | 7 | <br>

\hline\end{array}

| 4 |
| :--- |
| 0 |

$$
84 \div 2=42
$$

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Maths Written Calculation Policy

## Vocabulary

divide, share, group, divided by, equal sharing, equal groups, divisible by, remainder, quotient

First, I am sharing ... tens into ... equal groups.
There are ... tens in each group.
I have ... ten(s) remaining.
I need to regroup the remaining ... ten(s) into ... ones.
I now have ... ones in total.
Then, I am sharing ... ones into ... equal groups.
There are ... ones in each group.
I have ... one(s) remaining.
The quotient is ... with ... remainders.


4 hundreds shared into 3 equal groups requires 1 hundred to be regrouped into 10 tens.

## Vocabulary

divide, share, group, divided by, equal sharing, equal groups, divisible by, remainder, quotient

First, I am sharing ... hundreds into ... equal groups.
There are ... hundreds in each group.
I have ... hundred(s) remaining.
I need to regroup the remaining ... hundreds into ...tens.
I now have ... tens in total.
Next, I am sharing ... tens into .. equal groups.



| NC Statement: Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. <br> Short division for numbers up to 4-digits (grouping structure) |  |  |  |
| :---: | :---: | :---: | :---: |
| Concrete $\text { e.g. } 438 \div 3$ <br> "There is 1 group of hundreds without regrouping, 4 groups of tens and 6 groups of ones with no remainder" | Pictorial $\text { e.g. } 438 \div 3$ | Abstract $\begin{aligned} & \text { e.g. } 438 \div 3 \\ & 3 \longdiv { 1 ^ { 1 } 3 ^ { 1 } 8 } \end{aligned}$ $438 \div 3=146$ <br> This progresses to "How many 3s are in 438?" | $\square$ $\frac{5}{6}$ $\frac{0}{3}$ |
| Vocabulary divide, share, group, divided by, equal sharing, equal How many ... are in ... | ups, divisible by, remainder, quotient, | dividend |  |

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| NC Statement: Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context Long division for numbers up to 4 digits |  |  |  | $\frac{0}{\frac{5}{\mathbf{u}}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Concrete <br> e.g. $3016 \div 13$ <br> "How many groups of 13 thousands are in 3 thousand, without regrouping?" I can make zero groups of 13 thousand. There are 3 thousand remaining. I need to regroup the 3 thousands into 30 hundreds." | Pictorial <br> e.g. $3016 \div 13$ <br> Jottings: multiples of the divisor <br> 13 <br> 26 <br> 39 <br> 52 <br> 65 <br> 78 <br> 104 | Abstract <br> e.g. $3016 \div 13$ $3016 \div 13=232$ | "How many 13s in 3016?" |  |
| Vocabulary divide, share, group, divided by, equal sharing, equal How many ... are in ... | ups, divisible by, rem | dividend |  |  |

## Appendix A: Strategies for four operations (in children's planners)

## Examples of formal written methods for addition, subtraction, multiplication and division

## Addition and Subtraction

| $789+642$ becomes |
| ---: |
| 77 8 9 <br> + 6 4 |
| 1 | |  |  |  |
| ---: | :--- | :--- |
| 1 | 3 | 1 |
|  | 1 |  |

Answer: 1431

874-523 becomes

$$
\begin{array}{r}
874 \\
-\quad 523 \\
\hline 351 \\
\hline \text { Answer: } 351
\end{array}
$$

932-457 becomes
$83^{12} 2$
$-\quad 437$
475

Answer: 475

$$
\begin{aligned}
& 2741 \times 6 \text { becomes } \\
& \begin{array}{llll}
2 & 7 & 4 & 1
\end{array} \\
& \begin{array}{ccccc}
\times & & & & 6 \\
\hline 1 & 6 & 4 & 4 & 6 \\
\hline & 4 & 2 & &
\end{array} \\
& \text { Answer: } 16446
\end{aligned}
$$

## Long Multiplication

| $24 \times 16$ becomes | $124 \times 26$ becomes$\begin{array}{r} 124 \\ \times \quad 26 \\ \hline \end{array}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 6 | 3 | 2 | 1 |
| $\times 16$ |  | X |  |  |  | 1 | 5 |
| $1{ }^{1} x^{4} 4$ | $744$ |  | 3 | 11 | 61 | 0 | 5 |
| 2 4 0 <br> 3 8 4 | 2 4 8 0 <br> 3 2 2 4 | + | 6 | 3 | 2 | 1 | 0 |
|  | $\begin{array}{ll} \hline 1 & 1 \end{array}$ |  | 9 | 4 | 8 | 1 | 5 |
| Answer: 384 | Answer: 3224 |  |  | 1 |  |  |  |

Short Division
$98 \div 7$ becomes

$$
\begin{gathered}
1 \quad 4 \\
7 \longdiv { 9 ^ { 2 } 8 }
\end{gathered}
$$

Answer: 14

$$
432 \div 5 \text { becomes }
$$

$$
5 \longdiv { 4 3 ^ { 3 } 2 } \text { r2 }
$$

Answer: 86 remainder 2

## Short multiplication

## $24 \times 6$ becomes

$$
\begin{array}{r}
24 \\
\times \quad 6 \\
\hline 144 \\
\hline 2
\end{array}
$$

Answer: 144
$342 \times 7$ becomes

\[

\]

$124 \times 26$ becomes

Answer: 3224
$496 \div 11$ becomes
45 r 1

| 1 | 1 | 4 | 9 | 6 |
| :--- | :--- | :--- | :--- | :--- |

Answer: $45 \frac{1}{11}$

$$
432 \div 15 \text { becomes }
$$

Answer: 28.8

A Box of Knowledge can be written including the $1 x, 2 x, 5 x, 10 x$ facts to help each step.

